1 Equations

Assuming $Z_i = \frac{1}{2}\frac{Q''}{Q'}(z_i)$, $P_0 = N$, $P_1 = 0$, $D = \mathrm{discrim}(Q)$

$$D\frac{Q''}{Q'^2} = B_2$$

We define:

$$\begin{array}{cc} P_{2i} = \mathscr{P}(Q''^i B_2^i) & R_{2i} = \mathscr{P}(zQ''^i B_2^i) \\ P_{2i+1} = \mathscr{S}(Q''^{i+1} B_2^i) & R_{2i+1} = \mathscr{S}(zQ''^{i+1} B_2^i) \end{array}$$

where:

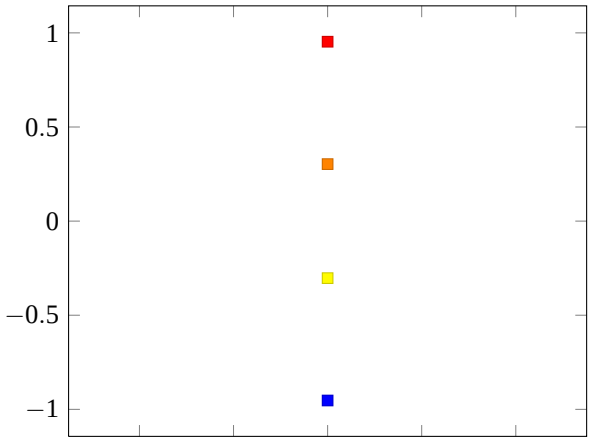
$$\begin{array}{c} \mathscr{P}(\sum_{j=0}^d a_j z^j) = \sum_{j=0}^d a_j p_j \\ \mathscr{S}(\sum_{j=0}^d a_j z^j) = \sum_{j=0}^d a_j h_{j-N+1} \end{array}$$

where p_k (resp. h_k) is the k-th power sum (resp. k-th complete sum) in $z_1, \ldots, z_N$.

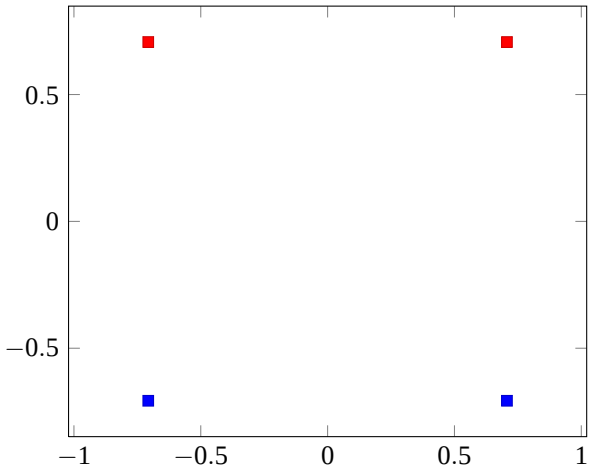
We have the equations:

$$\begin{array}{l} R_2 = 0 \\ R_3 - (2N-3)P_2 = 0 \\ R_4 - (2N-4)DP_3 = 0 \\ R_5 - P_2^2 - (2N-5)P_4 = 0 \\ R_6 - 2DP_2P_3 - (2N-6)DP_5 = 0 \\ R_7 - DP_3^2 - 2P_2P_4 - (2N-7)P_6 = 0 \\ R_8 - 2DP_3P_4 - 2DP_2P_5 - (2N-8)DP_7 = 0 \\ R_9 - P_4^2 - 2DP_3P_5 - 2P_2P_6 - (2N-9)P_8 = 0 \\ R_{10} - 2DP_4P_5 - 2DP_3P_6 - 2DP_2P_7 - (2N-10)DP_9 = 0 \end{array}$$

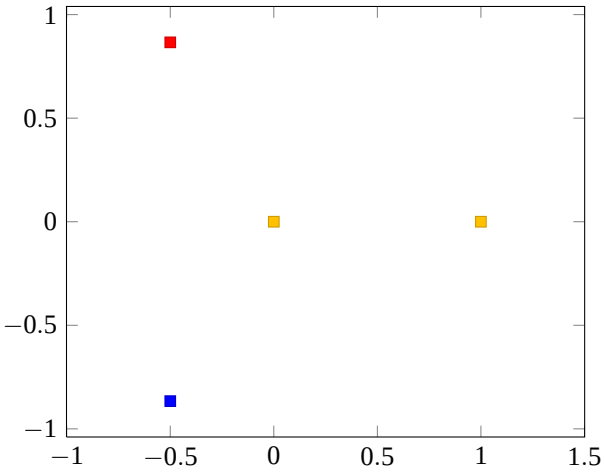
2 When $n = 4$



$[e2-1, 12*e4-1, e3]$ and degree 1 in $e3$
$z_1 = -.9530206139\,i$
$z_2 = -.3029054465\,i$
$z_3 = .3029054465\,i$
$z_4 = .9530206139\,i$

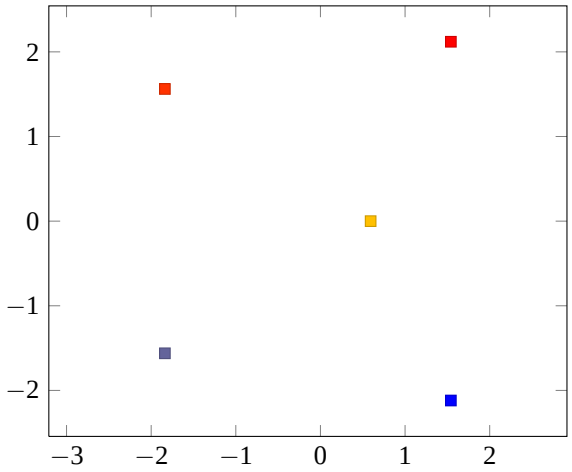


$[e2, e3, e4-1]$ and degree 1 in $e4$
$z_1 = -.7071067812 - .7071067812\,i$
$z_2 = -.7071067812 + .7071067812\,i$
$z_3 = .7071067812 - .7071067812\,i$
$z_4 = .7071067812 + .7071067812\,i$

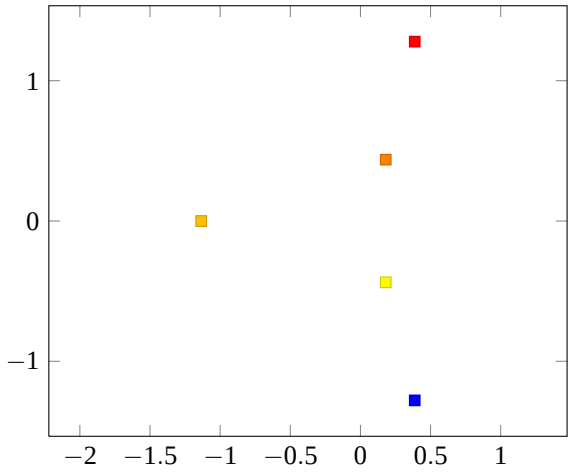


$[e2, e3-1, e4]$ and degree 1 in $e4$
$z_1 = -.5000000000 - .8660254038\,i$
$z_2 = -.5000000000 + .8660254038\,i$
$z_3 = 0.\,i$
$z_4 = 1.$

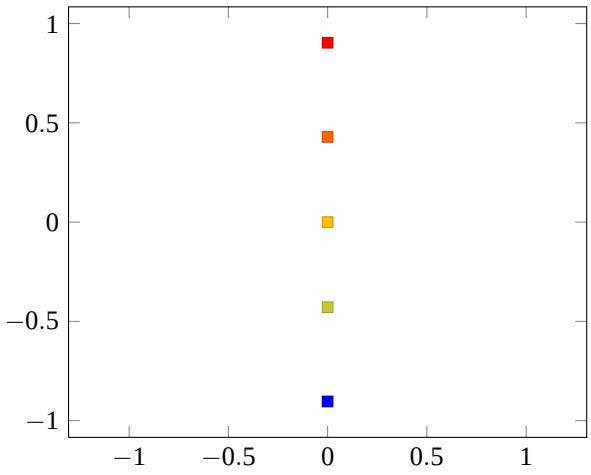
3 When $n = 5$



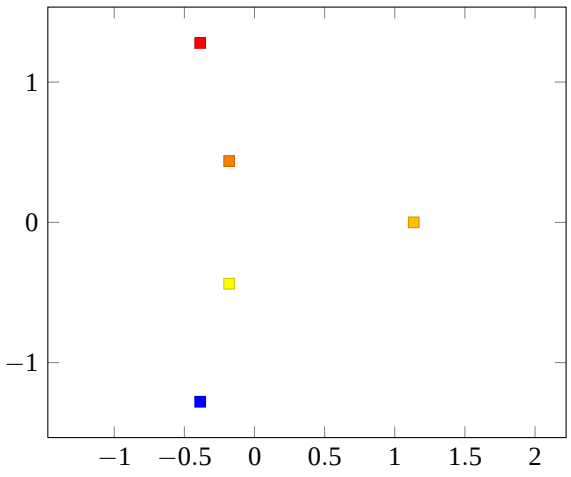
$[e2-1]$ and degree 17 in $e3$
$z_1 = -1.837874133 - 1.561394409\,i$
$z_2 = -1.837874133 + 1.561394409\,i$
$z_3 = .5931170471$
$z_4 = 1.541315610 - 2.119286803\,i$
$z_5 = 1.541315610 + 2.119286803\,i$



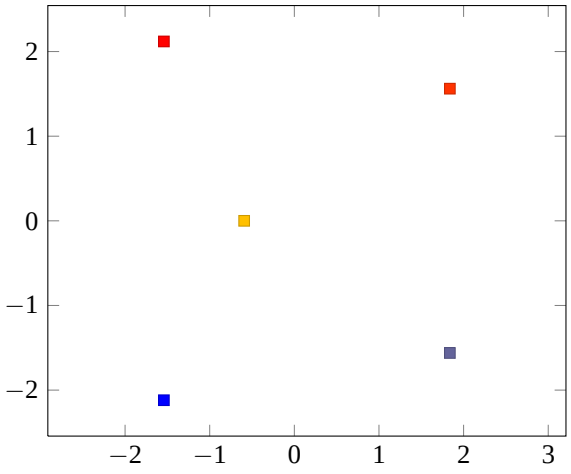
$[e2-1]$ and degree 17 in $e3$
$z_1 = -1.135180207$
$z_2 = .1805290326 - .4373777064\,i$
$z_3 = .1805290326 + .4373777064\,i$
$z_4 = .3870610709 - 1.278837283\,i$
$z_5 = .3870610709 + 1.278837283\,i$



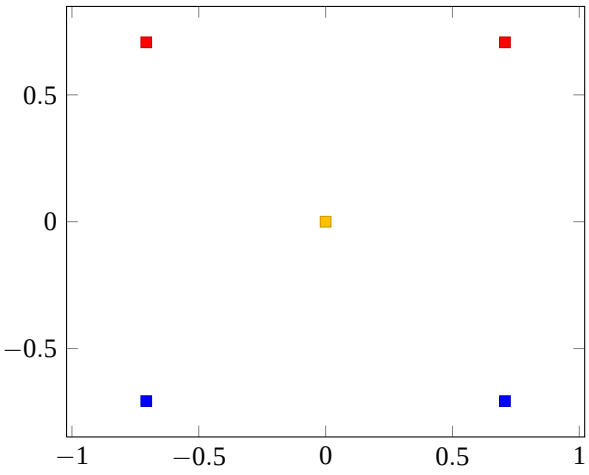
$[e2-1]$ and degree 17 in $e3$
$z_1 = -.9034532451\,i$
$z_2 = -.4286866384\,i$
$z_3 = 0.\,i$
$z_4 = .4286866384\,i$
$z_5 = .9034532451\,i$



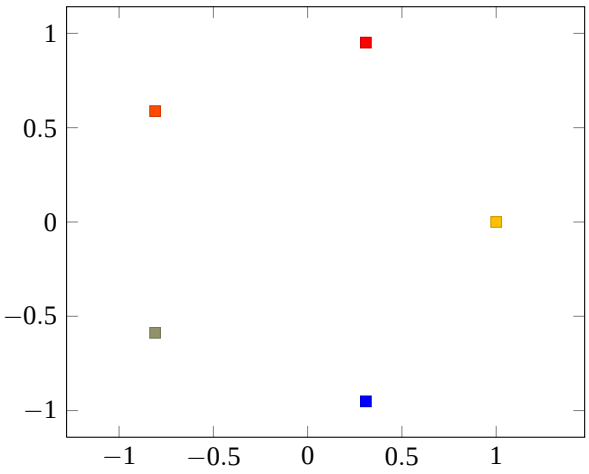
$[e2-1]$ and degree 17 in $e3$
$z_1 = -.3870610709 - 1.278837283\,i$
$z_2 = -.3870610709 + 1.278837283\,i$
$z_3 = -.1805290326 - .4373777064\,i$
$z_4 = -.1805290326 + .4373777064\,i$
$z_5 = 1.135180207$



$[e2-1]$ and degree 17 in $e3$
$z_1 = -1.541315610 - 2.119286803\,i$
$z_2 = -1.541315610 + 2.119286803\,i$
$z_3 = -.5931170471$
$z_4 = 1.837874133 - 1.561394409\,i$
$z_5 = 1.837874133 + 1.561394409\,i$

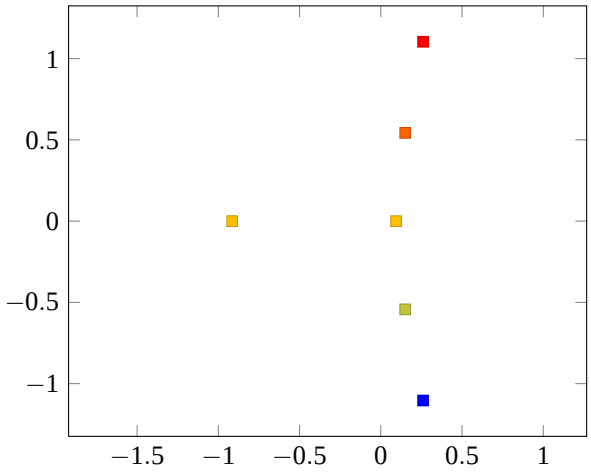


$[e2, e3, e4-1, e5]$ and degree 1 in $e5$
$z_1 = -.7071067812 - .7071067812\,i$
$z_2 = -.7071067812 + .7071067812\,i$
$z_3 = 0.\,i$
$z_4 = .7071067812 - .7071067812\,i$
$z_5 = .7071067812 + .7071067812\,i$

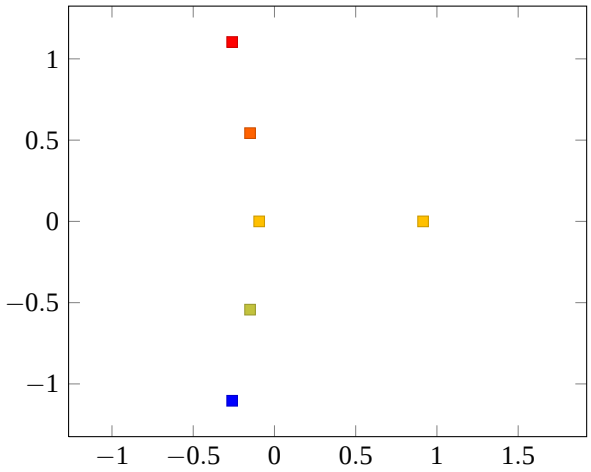


$[e2, e5-1, e4, e3]$ and degree 1 in $e3$
$z_1 = -.8090169944 - .5877852523\,i$
$z_2 = -.8090169944 + .5877852523\,i$
$z_3 = .3090169944 - .9510565163\,i$
$z_4 = .3090169944 + .9510565163\,i$
$z_5 = 1.$

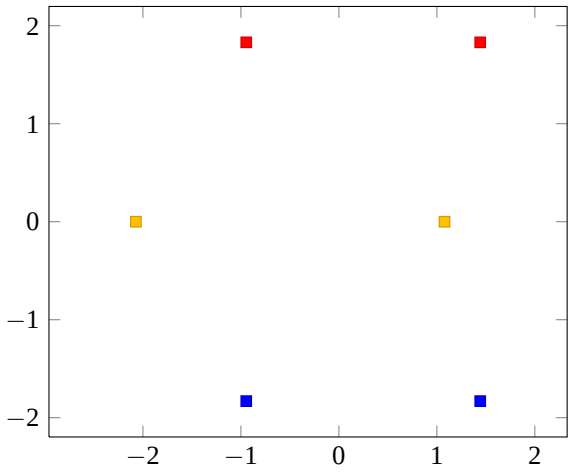
4 When $n = 6$



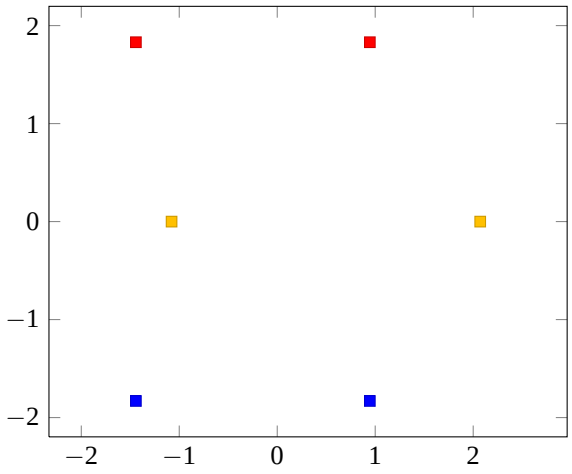
$[e2-1]$ and degree 22 in $e3$
$z_1 = -.9147704148$
$z_2 = .9402485960e - 1$
$z_3 = .1501130018 - .5430424944\,i$
$z_4 = .1501130018 + .5430424944\,i$
$z_5 = .2602597758 - 1.103719487\,i$
$z_6 = .2602597758 + 1.103719487\,i$



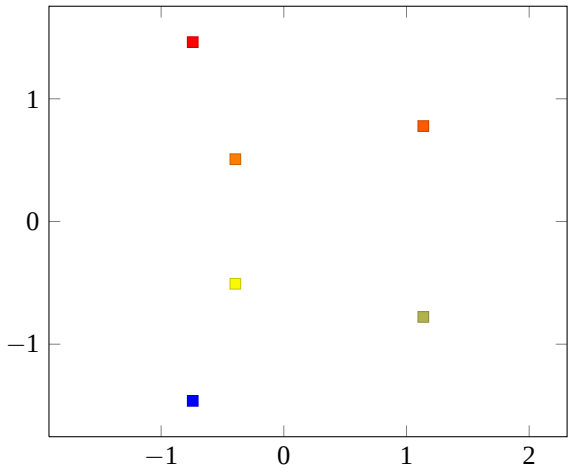
$[e2-1]$ and degree 22 in $e3$
$z_1 = -.2602597758 - 1.103719487\,i$
$z_2 = -.2602597758 + 1.103719487\,i$
$z_3 = -.1501130018 - .5430424944\,i$
$z_4 = -.1501130018 + .5430424944\,i$
$z_5 = -.9402485960e - 1$
$z_6 = .9147704148$



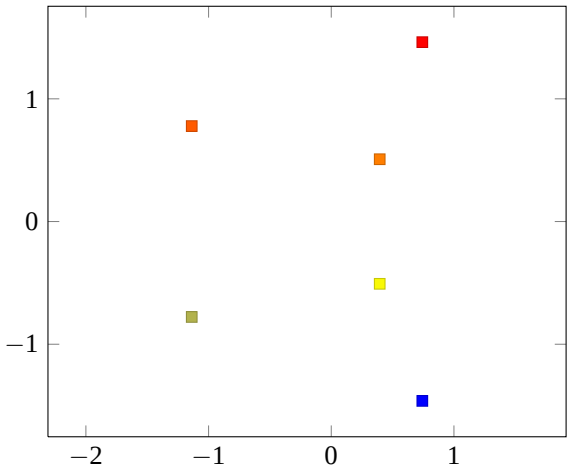
$[e2-1]$ and degree 4 in $e3$
$z_1 = -2.071760264$
$z_2 = -.9455170461 - 1.830513878\,i$
$z_3 = -.9455170461 + 1.830513878\,i$
$z_4 = 1.078239639$
$z_5 = 1.442277358 - 1.830513878\,i$
$z_6 = 1.442277358 + 1.830513878\,i$



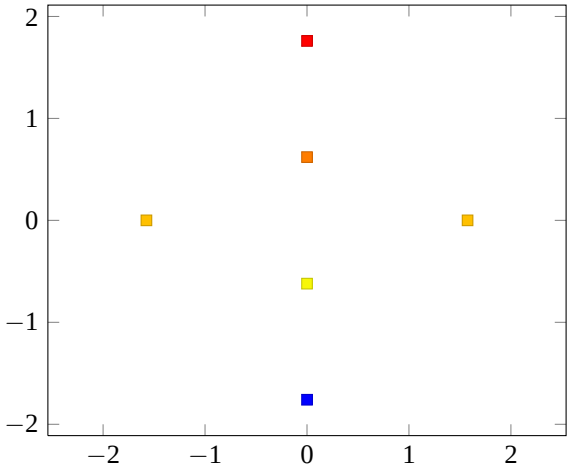
$[e2-1]$ and degree 4 in $e3$
$z_1 = -1.442277358 - 1.830513878\,i$
$z_2 = -1.442277358 + 1.830513878\,i$
$z_3 = -1.078239639$
$z_4 = .9455170461 - 1.830513878\,i$
$z_5 = .9455170461 + 1.830513878\,i$
$z_6 = 2.071760264$



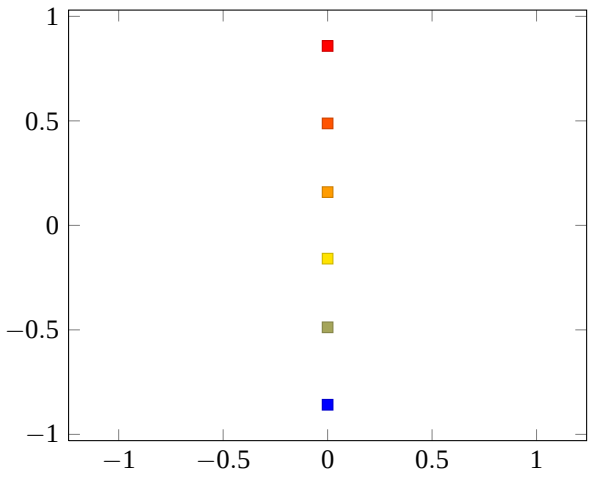
$[e2-1,9*e6-19,3*e5+8,e4-3,3*e3-8]$ and degr
$z_1 = -.7422271990 - 1.461902200\,i$
$z_2 = -.7422271990 + 1.461902200\,i$
$z_3 = -.3949308436 - .5077133059\,i$
$z_4 = -.3949308436 + .5077133059\,i$
$z_5 = 1.137158043 - .7778619134\,i$
$z_6 = 1.137158043 + .7778619134\,i$



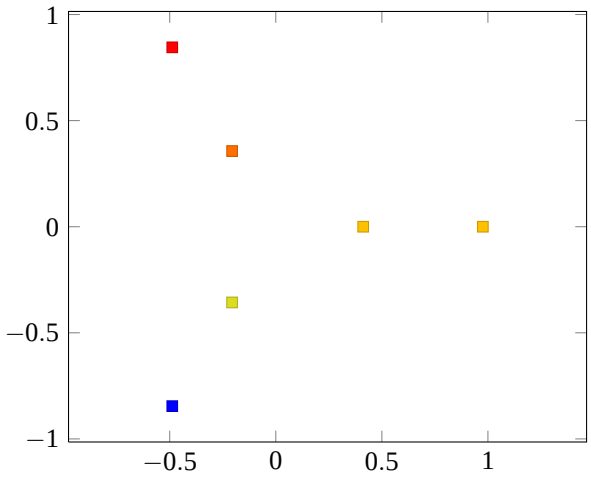
$[e2-1,9e6-19,3e5-8,e4-3,3e3+8]$ and degree 1
$z_1 = -1.137158043 - .7778619134\,i$
$z_2 = -1.137158043 + .7778619134\,i$
$z_3 = .3949308436 - .5077133059\,i$
$z_4 = .3949308436 + .5077133059\,i$
$z_5 = .7422271990 - 1.461902200\,i$
$z_6 = .7422271990 + 1.461902200\,i$



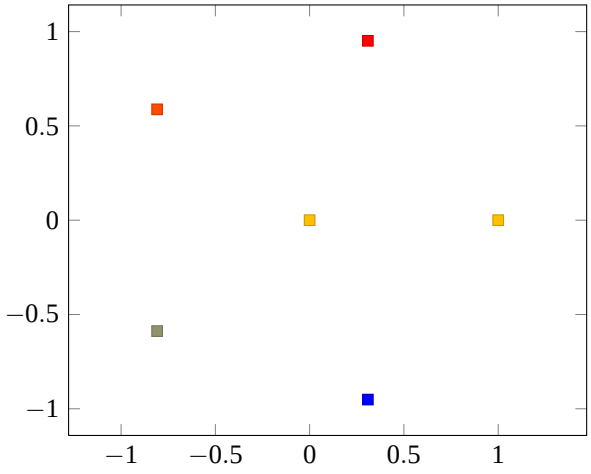
$[e2-1,e5,e3]$ and degree 6 in $e4$
$z_1 = -1.574999952$
$z_2 = -1.759393245\,i$
$z_3 = -.6206128082\,i$
$z_4 = .6206128082\,i$
$z_5 = 1.759393245\,i$
$z_6 = 1.574999952$



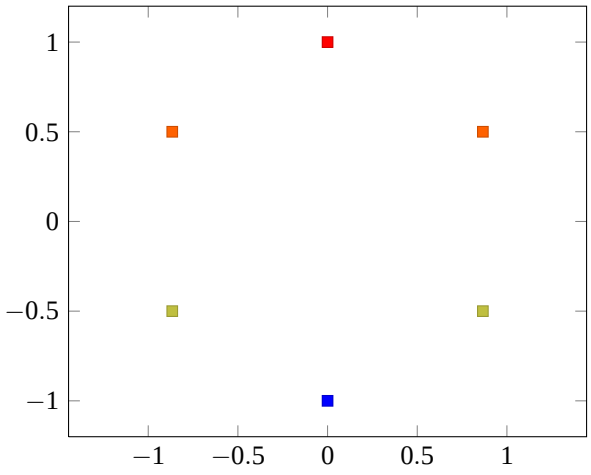
$[e2-1,e5,e3]$ and degree 6 in $e4$
$z_1 = -.8583195786\,i$
$z_2 = -.4877831142\,i$
$z_3 = -.1592329569\,i$
$z_4 = .1592329569\,i$
$z_5 = .4877831142\,i$
$z_6 = .8583195786\,i$



$[e2, e3-1, e5, e4]$ and degree 2 in $e6$
$z_1 = -.4880610968 - .8453466169\,i$
$z_2 = -.4880610968 + .8453466169\,i$
$z_3 = -.2060020188 - .3568059630\,i$
$z_4 = -.2060020188 + .3568059630\,i$
$z_5 = .4120040376$
$z_6 = .9761221936$

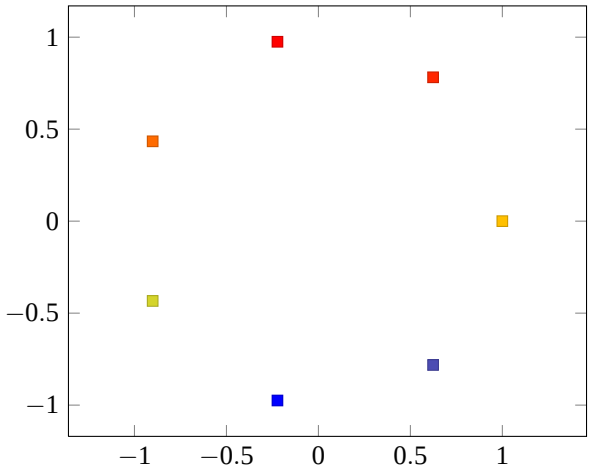


$[e2, e3, e4, e6, e5-1]$ and degree 1 in $e5$
$z_1 = -.8090169944 - .5877852523\,i$
$z_2 = -.8090169944 + .5877852523\,i$
$z_3 = 0.\,i$
$z_4 = .3090169944 - .9510565163\,i$
$z_5 = .3090169944 + .9510565163\,i$
$z_6 = 1.$

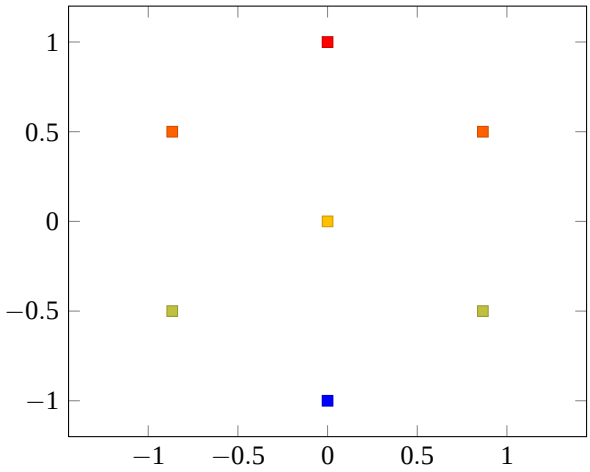


$[e2, e3, e4, e6-1, e5]$ and degree 1 in $e5$
$z_1 = -.8660254038 - .5000000000\,i$
$z_2 = -.8660254038 + .5000000000\,i$
$z_3 = -1.000000000\,i$
$z_4 = 1.000000000\,i$
$z_5 = .8660254038 - .5000000000\,i$
$z_6 = .8660254038 + .5000000000\,i$

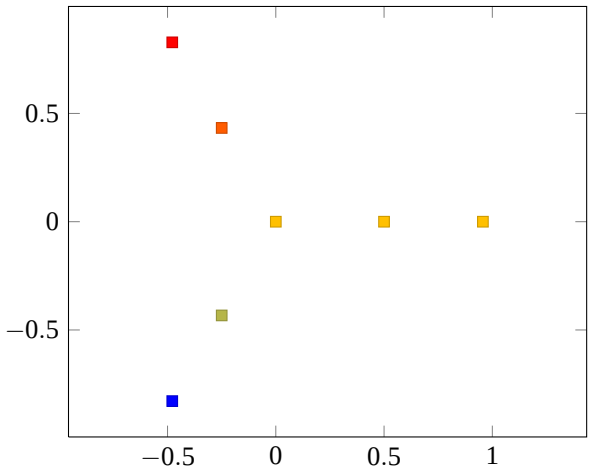
5 When $n = 7$



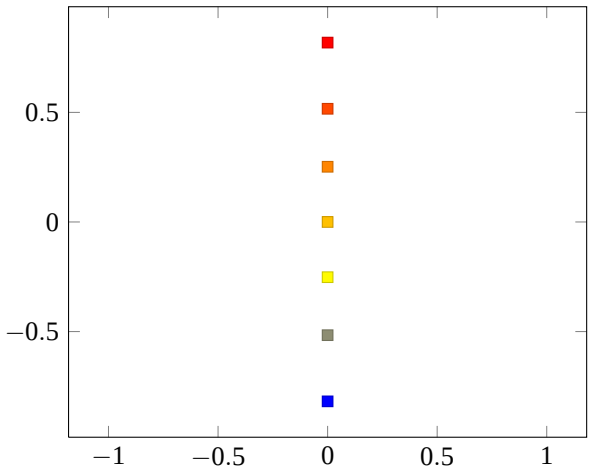
$[e2, e3, e4, e5, e7-1, e6]$ and degree 1 in $e6$
$z_1 = 1.000000000$
$z_2 = 0.6234898018 + 0.7818314825\,i$
$z_3 = 0.6234898018 - 0.7818314825\,i$
$z_4 = -0.9009688679 + 0.4338837391\,i$
$z_5 = -0.9009688679 - 0.4338837391\,i$
$z_6 = -0.2225209340 + 0.9749279122\,i$
$z_7 = -0.2225209340 - 0.9749279122\,i$



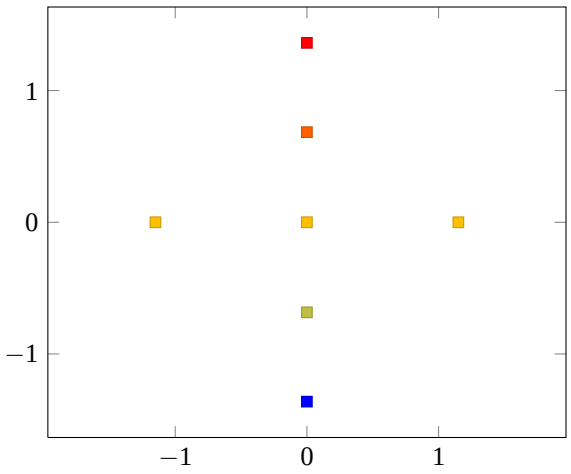
$[e2, e3, e4, e5, e7, e6-1]$ and degree 1 in $e6$
$z_1 = 0.0000000000\,i$
$z_2 = 0.8660254038 - 0.5000000000\,i$
$z_3 = 0.8660254038 + 0.5000000000\,i$
$z_4 = -0.8660254038 + 0.5000000000\,i$
$z_5 = -0.8660254038 - 0.5000000000\,i$
$z_6 = 1.000000000\,i$
$z_7 = -1.000000000\,i$



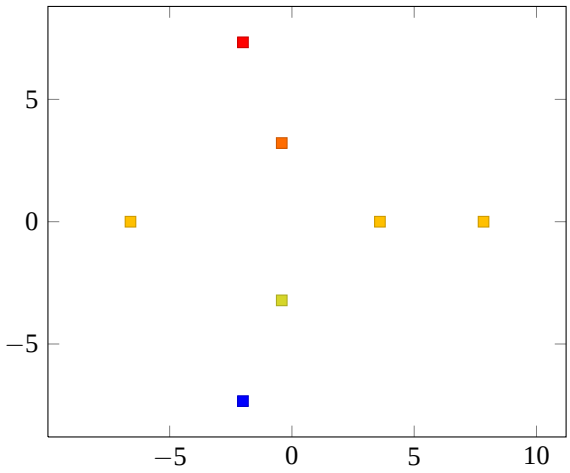
$[e2, e3-1, e7, e5, e4]$ and degree 2 in $e6$
$z_1 = 0.0000000000\,i$
$z_2 = 0.4997476001$
$z_3 = 0.9565345264$
$z_4 = -0.4782672632 + 0.8283831995\,i$
$z_5 = -0.4782672632 - 0.8283831995\,i$
$z_6 = -0.2498738000 + 0.4327941172\,i$
$z_7 = -0.2498738000 - 0.4327941172\,i$



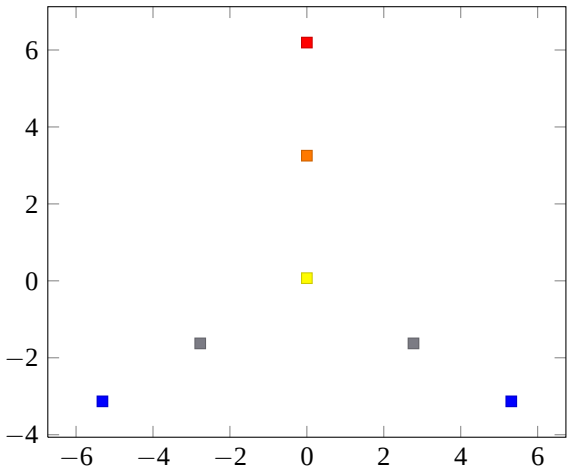
$[e2-1, e3, e7, e5]$ and degree 6 in $e4$
$z_1 = 0.0000000000\,i$
$z_2 = 0.5164692453\,i$
$z_3 = -0.5164692453\,i$
$z_4 = 0.8184130426\,i$
$z_5 = -0.8184130426\,i$
$z_6 = 0.2519119097\,i$
$z_7 = -0.2519119097\,i$



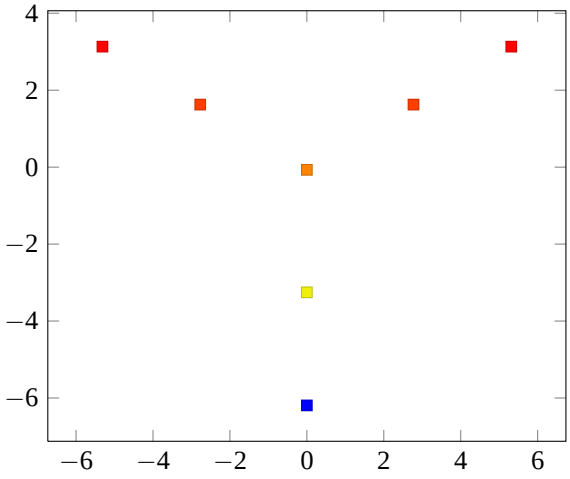
$[e2-1, e3, e7, e5]$ and degree 6 in $e4$
$z_1 = 0.0000000000\,i$
$z_2 = 0.6840809939\,i$
$z_3 = -0.6840809939\,i$
$z_4 = 1.361938935\,i$
$z_5 = -1.361938935\,i$
$z_6 = 1.150149759$
$z_7 = -1.150149759$



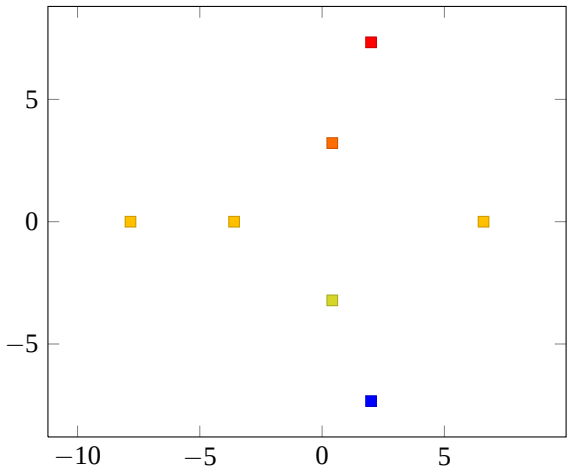
$[e2-1]$ and degree 82 in $e3$
$z_1 = 3.598860894$
$z_2 = 7.834872130$
$z_3 = -2.000847242 - 7.333743175\,i$
$z_4 = -2.000847242 + 7.333743175\,i$
$z_5 = -0.4157544708 - 3.216280228\,i$
$z_6 = -0.4157544708 + 3.216280228\,i$
$z_7 = -6.600529599$



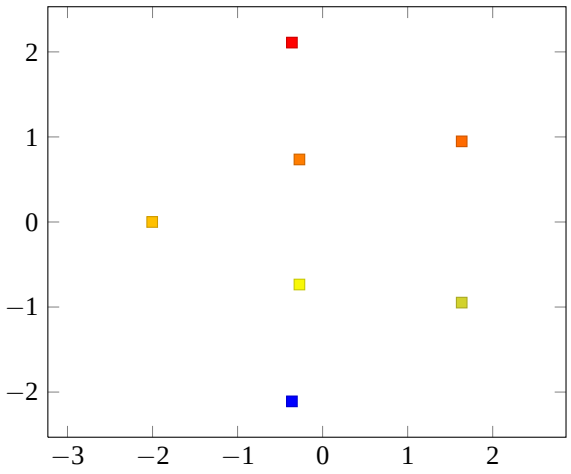
$[e2-1]$ and degree 82 in $e3$
$z_1 = 0.06990470615\,i$
$z_2 = 2.772094332 - 1.625832642\,i$
$z_3 = 5.314078748 - 3.132170522\,i$
$z_4 = -5.314078748 - 3.132170522\,i$
$z_5 = -2.772094332 - 1.625832642\,i$
$z_6 = 6.192902391\,i$
$z_7 = 3.253199231\,i$



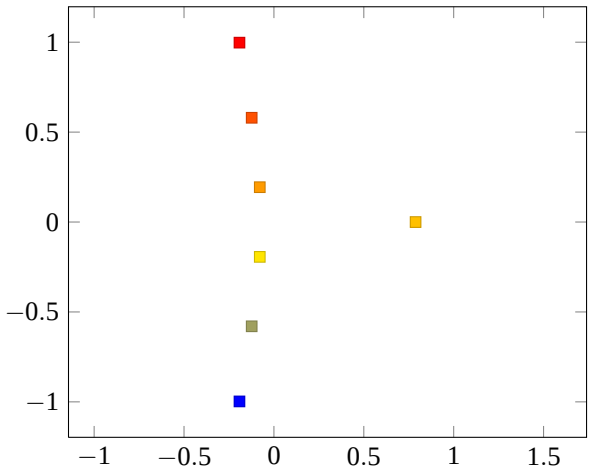
$[e2-1]$ and degree 82 in $e3$
$z_1 = -0.06990470615\,i$
$z_2 = 2.772094332 + 1.625832642\,i$
$z_3 = 5.314078748 + 3.132170522\,i$
$z_4 = -6.192902391\,i$
$z_5 = -3.253199231\,i$
$z_6 = -2.772094332 + 1.625832642\,i$
$z_7 = -5.314078748 + 3.132170522\,i$



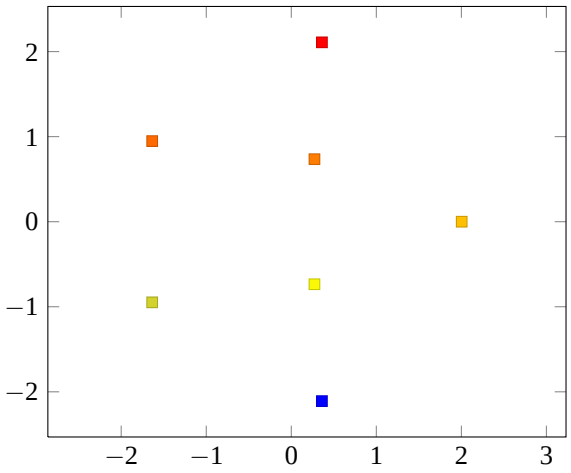
$[e2-1]$ and degree 82 in $e3$
$z_1 = -7.834872130$
$z_2 = -3.598860894$
$z_3 = 0.4157544708 - 3.216280228\,i$
$z_4 = 0.4157544708 + 3.216280228\,i$
$z_5 = 2.000847242 - 7.333743175\,i$
$z_6 = 2.000847242 + 7.333743175\,i$
$z_7 = 6.600529599$



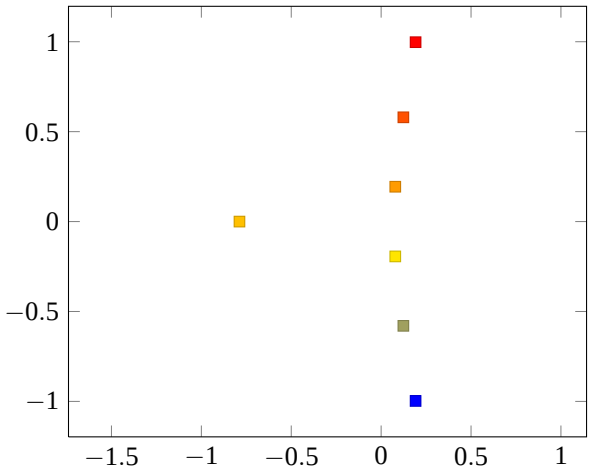
$[e2-1]$ and degree 64 in $e3$
$z_1 = 1.635499617 - 0.9479855871\,i$
$z_2 = 1.635499617 + 0.9479855871\,i$
$z_3 = -0.3607761542 - 2.109129928\,i$
$z_4 = -0.3607761542 + 2.109129928\,i$
$z_5 = -2.004161219$
$z_6 = -0.2726428532 - 0.7352400466\,i$
$z_7 = -0.2726428532 + 0.7352400466\,i$



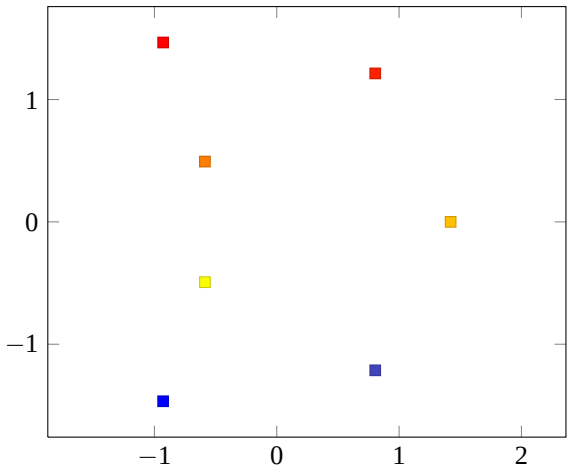
$[e2-1]$ and degree 64 in $e3$
$z_1 = -0.1236891893 + 0.5797664286\,i$
$z_2 = -0.1236891893 - 0.5797664286\,i$
$z_3 = -0.1916429057 + 0.9974964946\,i$
$z_4 = -0.1916429057 - 0.9974964946\,i$
$z_5 = -0.07865618380 + 0.1937471886\,i$
$z_6 = -0.07865618380 - 0.1937471886\,i$
$z_7 = 0.7879765575$



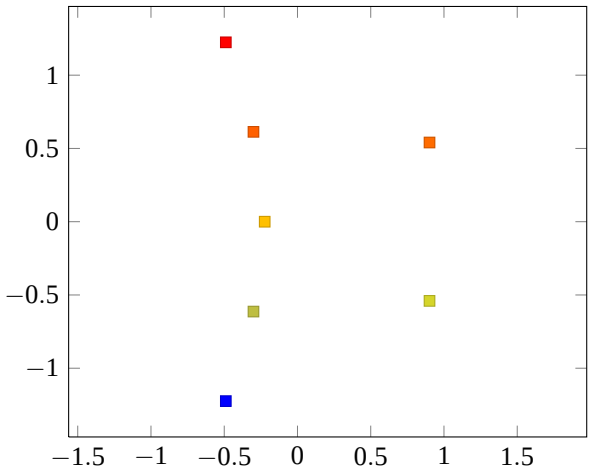
$[e2-1]$ and degree 64 in $e3$
$z_1 = 0.3607761542 - 2.109129928\,i$
$z_2 = 0.3607761542 + 2.109129928\,i$
$z_3 = -1.635499617 - 0.9479855871\,i$
$z_4 = -1.635499617 + 0.9479855871\,i$
$z_5 = 0.2726428532 - 0.7352400466\,i$
$z_6 = 0.2726428532 + 0.7352400466\,i$
$z_7 = 2.004161219$



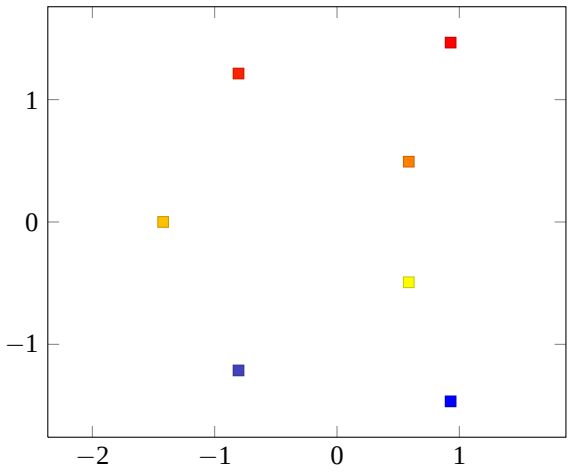
$[e2-1]$ and degree 64 in $e3$
$z_1 = 0.1236891893 + 0.5797664286\,i$
$z_2 = 0.1236891893 - 0.5797664286\,i$
$z_3 = 0.1916429057 + 0.9974964946\,i$
$z_4 = 0.1916429057 - 0.9974964946\,i$
$z_5 = 0.07865618380 + 0.1937471886\,i$
$z_6 = 0.07865618380 - 0.1937471886\,i$
$z_7 = -0.7879765575$



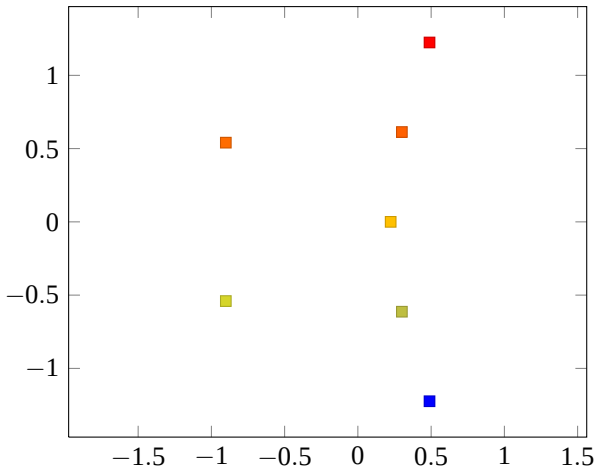
$[e2-1]$ and degree 64 in $e3$
$z_1 = 1.421276379$
$z_2 = 0.8050059509 + 1.213435326\,i$
$z_3 = 0.8050059509 - 1.213435326\,i$
$z_4 = -0.9287769292 + 1.466185554\,i$
$z_5 = -0.9287769292 - 1.466185554\,i$
$z_6 = -0.5868672112 + 0.4929119525\,i$
$z_7 = -0.5868672112 - 0.4929119525\,i$



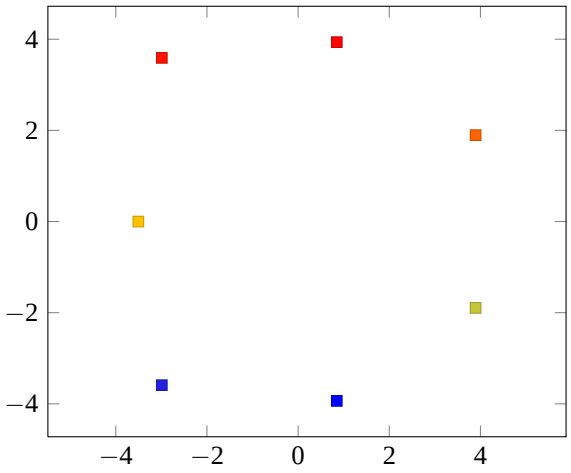
$[e2-1]$ and degree 64 in $e3$
$z_1 = 0.9013547555 + 0.5405057884\,i$
$z_2 = 0.9013547555 - 0.5405057884\,i$
$z_3 = -0.4890637549 + 1.224227790\,i$
$z_4 = -0.4890637549 - 1.224227790\,i$
$z_5 = -0.3004067774 + 0.6132079202\,i$
$z_6 = -0.3004067774 - 0.6132079202\,i$
$z_7 = -0.2237684465$



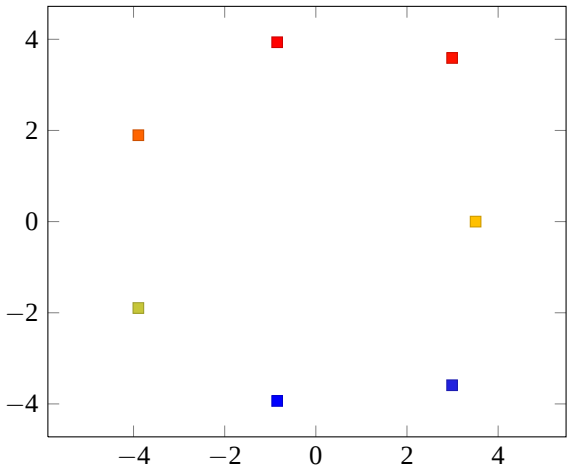
$[e2-1]$ and degree 64 in $e3$
$z_1 = -1.421276379$
$z_2 = -0.8050059509 + 1.213435326\,i$
$z_3 = -0.8050059509 - 1.213435326\,i$
$z_4 = 0.5868672112 + 0.4929119525\,i$
$z_5 = 0.5868672112 - 0.4929119525\,i$
$z_6 = 0.9287769292 + 1.466185554\,i$
$z_7 = 0.9287769292 - 1.466185554\,i$



$[e2-1]$ and degree 64 in $e3$
$z_1 = -0.9013547555 + 0.5405057884\,i$
$z_2 = -0.9013547555 - 0.5405057884\,i$
$z_3 = 0.3004067774 + 0.6132079202\,i$
$z_4 = 0.3004067774 - 0.6132079202\,i$
$z_5 = 0.4890637549 + 1.224227790\,i$
$z_6 = 0.4890637549 - 1.224227790\,i$
$z_7 = 0.2237684465$



$[e2-1]$ and degree 64 in $e3$
$z_1 = 3.894124852 - 1.895667401\,i$
$z_2 = 3.894124852 + 1.895667401\,i$
$z_3 = 0.8485714142 - 3.935484384\,i$
$z_4 = 0.8485714142 + 3.935484384\,i$
$z_5 = -3.505433095$
$z_6 = -2.989979719 - 3.589804854\,i$
$z_7 = -2.989979719 + 3.589804854\,i$



$[e2-1]$ and degree 64 in $e3$
$z_1 = 2.989979719 - 3.589804854\,i$
$z_2 = 2.989979719 + 3.589804854\,i$
$z_3 = 3.505433095$
$z_4 = -0.8485714142 - 3.935484384\,i$
$z_5 = -0.8485714142 + 3.935484384\,i$
$z_6 = -3.894124852 - 1.895667401\,i$
$z_7 = -3.894124852 + 1.895667401\,i$

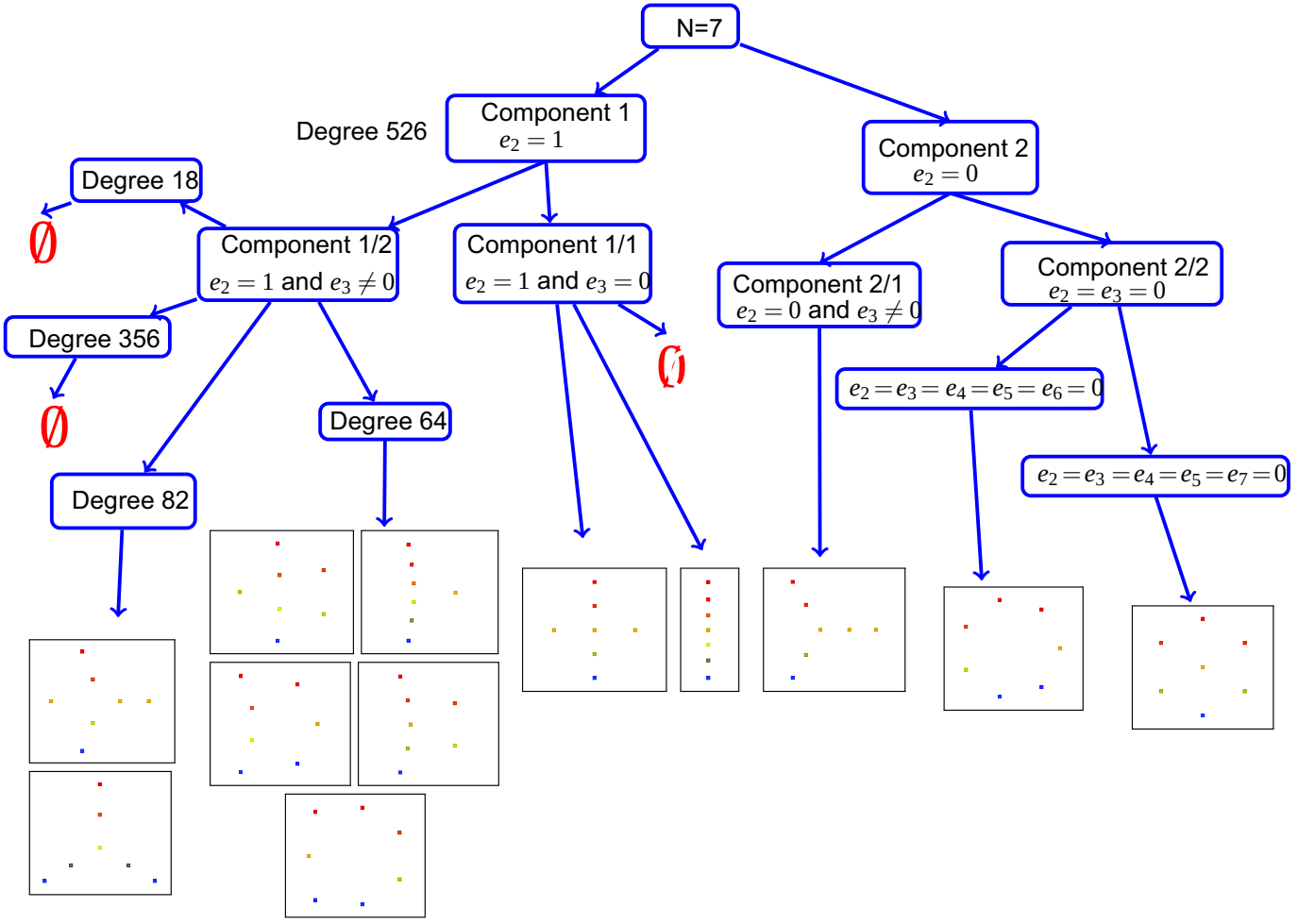


Figure 1: All the solutions of the vortices problem when $N = 7$